Interannual variability
in the North Atlantic SST and wind forcing

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IRI Seminar
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Outline

1 Methodology
   - Multivariate Singular Spectrum Analysis
   - Varimax rotation of ST-EOFs
   - Significance test
   - Compression onto a few PCs

2 Interannual variability in the North Atlantic
   - The SODA reanalysis
   - Low-frequency variability in the double-gyre circulation
   - Interannual variability in SODA
   - Intrinsic vs. external variation
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Multivariate Singular Spectrum Analysis

**Idea:** Reconstruct dynamics from time-delayed embedding (Mañé-Takens)

**Data:** Multivariate time series $\mathbf{x}(n) = \{x_d(n)\}$

$d = 1 \ldots D$ channels; length $n = 1 \ldots N$

**Temporal embedding (T):** Window length $M$

$$
\mathbf{X}_d = \begin{pmatrix}
  x_d(1) & x_d(2) & \ldots & x_d(M) \\
  x_d(2) & x_d(3) & \ldots & x_d(M+1) \\
  \vdots & \vdots & \ddots & \vdots \\
  x_d(N') & x_d(N' + 1) & \ldots & x_d(N)
\end{pmatrix}
$$

reduced length $N' = N - M + 1$

**Spatial embedding (S):** Concatenate all channels; size $N \times D \cdot M$

$$
\mathbf{X} = \begin{pmatrix}
  \mathbf{X}_1 & \mathbf{X}_2 & \ldots & \mathbf{X}_D
\end{pmatrix}
$$
Multivariate Singular Spectrum Analysis

Extract information from \( X = \begin{pmatrix} X_1 & X_2 & \ldots & X_D \end{pmatrix} \) via Principal Component Analysis (PCA)

1. Compute covariance matrix \( C = X'X \)
2. Eigendecomposition \( \Lambda = E'CE \) gives
   - diagonal matrix \( \Lambda \) of eigenvalues \( \lambda_k \) (variance)
   - orthogonal matrix \( E \) of eigenvectors \( e_k \) (ST-EOFs)
3. Projection \( A = XE \) gives principal components (PCs)
4. Reconstruction \( AKE' = X_k \) gives reconstructed components (RCs)

\( (\text{Broomhead\&King 1986; Vautard\&Ghil 1989; Ghil et al., Reviews of Geophysics 2002}) \)

SSA toolkit at http://www.atmos.ucla.edu/tcd/ssa
Time series: composite of 4 harmonic oscillations + AR(1) noise
General problem of M-SSA and PCA: degenerate eigenvalues $\rightarrow$ mixture of eigenvectors

Solution: modified varimax rotation of ST-EOFs  \((Groth&Ghil, PRE, 2011)\)
Varimax rotation of ST-EOFs

- Improved separation between oscillations
- Unimodal eigenvectors without frequency mixing
- Simplified physical interpretation
Varimax rotation of ST-EOFs

Bivariate M-SSA analysis of NAO and SOI indices

▶ Absence of rotation → spurious correlations

*(Feliks, Groth, Robertson & Ghil, J. Climate 2013)*
Deterministic oscillations ↔ stochastic fluctuations?

Karhunen-Loève theorem: Spectral decomposition of stochastic process
Deterministic oscillations $\leftrightarrow$ stochastic fluctuations?

Karhunen-Loève theorem: Spectral decomposition of stochastic process
Significance test

Monte Carlo type significance test of eigenvalues

**Surrogate data:** \( \mathbf{x}_S \rightarrow \text{embedding} \ \mathbf{X}_S \); e.g. from AR(1) process

**Covariance matrix:** \( \mathbf{C}_S = \mathbf{X}'_S \mathbf{X}_S \)

**Projection:** \( \Lambda_P = \mathbf{E}' \mathbf{C}_S \mathbf{E} \)

**Comparison:** \( \Lambda = \mathbf{E}' \mathbf{C} \ \mathbf{E} \)

*(Allen&Smith, J. Climate 1996)*
Monte Carlo type significance test of eigenvalues

Significance test

Surrogate data: $x_S \rightarrow$ embedding $X_S$; e.g. from AR(1) process

Covariance matrix: $C_S = X'_S X_S$

Projection: $\Lambda_P = E' C_S E$

Comparison: $\Lambda = E' C E$  

(Allen & Smith, J. Climate 1996)
Significance test

**Idea:** Match eigenvectors & eigenvalues

### Improved significance test

**Surrogate data:** $x_S \rightarrow$ embedding $X_S$

**Covariance matrix:** $C_S = X'_S X_S$

**SSA:** $\Lambda_S = E'_S C_S E_S \quad \leftrightarrow \quad \Lambda = E' C E$

**Orthogonal rotation:** $T$ that best matches $E_S \Sigma_S T$ with $E \Sigma$

$(\Sigma = \Lambda^{1/2}$ singular values$)$

**Procrustes solution:**

- Singular Value Decomposition (SVD)
  
  $(E_S \Sigma_S)' E \Sigma = U S V'$

- $T = U V'$

**Comparison:** $\Lambda_T = T' \Lambda_S T \quad \leftrightarrow \quad \Lambda$

**Note:** classical test $\Lambda_P = E' C_S E \equiv E' E_S \Lambda_S E'_S E \equiv \tilde{T}' \Lambda_S \tilde{T}$

compares only eigenvectors
Deterministic oscillations $\leftrightarrow$ stochastic fluctuations?
Significance test

- Classical significance test
- New significance test

Deterministic oscillations $\leftrightarrow$ stochastic fluctuations?

- Discriminant power substantially improved
Significance test

Sensitivity vs. specificity

N = 250 | M = 40

# false positives (FP) | # true positives (TP)

--- classical test

--- improved test

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Interannual variability

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Significance test

Artificial variance compression

(a) Specificity

(b) Distribution of true oscillations

Classical test: high-rank eigenvalues are more likely to be significant

Improved test: Specificity in high-rank eigenvalues increased
Compression onto a few PCs

**Problem:** Analysis of high-dimensional data; $D \gg N$

**Approach:** Compression onto S-EOFs from conventional PCA analysis

- **(a)** Noise-free reference
- **(b)** $L = 3$, $\text{Var} = 49\%$, $\langle k \rangle = 5$, RMS = 0.66
- **(c)** $L = 10$, $\text{Var} = 86\%$, $\langle k \rangle = 3$, RMS = 0.30
- **(d)** $L = 130$, $\text{Var} = 100\%$, $\langle k \rangle = 2$, RMS = 0.20

- M-SSA ST-EOFs are biased toward S-EOFs at strong compression
- In case of three PCs, only standing waves are extracted
Compression onto a few PCs

**Problem:** Analysis of high-dimensional data; $D \gg N$

**Approach:** Compression onto S-EOFs from conventional PCA analysis

- Sensitivity shrinks at strong compression

![Graph showing the fraction of variance and number of TP/FP rotations against L for different datasets and compression levels.](image)
Methodology — Conclusions

1. Varimax rotation of ST-EOFs improves frequency separation
2. Procrustes rotation improves discriminant power of significance test
3. M-SSA ST-EOFs are less biased toward S-EOFs for weak/no compression
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Simple Ocean Data Assimilation

- SODA reanalysis of ocean climate variability, version 2.2.4
  \[ \text{(Giese and Ray 2011)} \]
- monthly dataset over the 137-yr interval 1871-2008
- Assimilation of hydrographic and SST data into a model of the global ocean forced with surface boundary conditions from the atmospheric 20CRv2 reanalysis
  \[ \text{(Compo et al. 2011)} \]

Dataset in this analysis

- Sea surface temperature (SST) \+ temperature at different depth
- Zonal surface wind stress (TAUX)
- Sea surface height (SSH)
- North Atlantic region
- Interannual activity $\Rightarrow$ annual subsampling (Chebyshev filter)
  \[ \text{(Feliks, Groth, Robertson & Ghil, J. Climate 2013)} \]
Low-frequency variability in the double-gyre circulation

- Mid-latitude, wind-driven ocean circulation
- Nonlinear effects break the symmetry; pitchfork bifurcation
- Time-dependent solution; alternation between stronger subtropical and stronger subpolar gyre \((\text{Jiang, Jin, Ghil, 1995})\)
- Interannual variability; e.g. displacement in Gulf Stream axis \((\text{Speich et al. 1995; Simonnet et al. 2003, 2005, 2009; Dijkstra & Ghil, 2005})\)
- 7–8-yr oscillation that involves the entire North Atlantic; e.g. M-SSA analysis of the UK Met Office SSTs in 1895-1994 interval — but only standing waves extracted due to strong compression \((\text{Moron et al. 1998})\)
- and many more
Low-frequency variability in the double-gyre circulation

- Observed variability in the basin-scale circulation due to
  - changes in the external atmospheric forcing or
  - system’s intrinsic instability and nonlinearity?
- Sharp SST gradients in Gulf Stream region can induce similar low-frequency variability in the overlying atmosphere \cite{Feliks2011}.
- **SODA**: atmosphere is boundary condition acting on the ocean model; possible intrinsic modes in the ocean cannot feed back to the atmospheric dynamics.
- **Question**: To which extent is the interannual variability in SST/SSH linked to TAUX?
Individual M-SSA analyses of

**TAUX**

![](image1)

**SST+SSH**

![](image2)

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Interannual variability

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Interannual variability in SODA

Joint M-SSA analysis of TAUX + SSH + SST

(a) TAUX | max = 0.004
(b) SSH | max = 0.014
(c) SST | max = 0.007
(d) TAUX | max = 0.004
(e) SSH | max = 0.018
(f) SST | max = 0.008
(g) TAUX | max = 0.006
(h) SSH | max = 0.008
(i) SST | max = 0.007

M-SSA indicates joint oscillatory modes; e.g. no frequency separation
Interannual variability in SODA

Phase composites in SST — 7.7-yr oscillation

<table>
<thead>
<tr>
<th>Period</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0 yr</td>
<td>-0.50</td>
<td>0.25</td>
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<tr>
<td>0.8 yr</td>
<td>-0.42</td>
<td>0.47</td>
</tr>
<tr>
<td>1.7 yr</td>
<td>-0.31</td>
<td>0.54</td>
</tr>
<tr>
<td>2.5 yr</td>
<td>-0.23</td>
<td>0.53</td>
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<tr>
<td>3.4 yr</td>
<td>-0.21</td>
<td>0.48</td>
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<tr>
<td>4.2 yr</td>
<td>-0.34</td>
<td>0.50</td>
</tr>
<tr>
<td>5.1 yr</td>
<td>-0.56</td>
<td>0.36</td>
</tr>
<tr>
<td>5.9 yr</td>
<td>-0.54</td>
<td>0.27</td>
</tr>
<tr>
<td>6.8 yr</td>
<td>-0.46</td>
<td>0.21</td>
</tr>
</tbody>
</table>

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Interannual variability in SODA

Phase composites in TAUX + SST — 7.7-yr oscillation

**TAUX:** Standing wave with strong in-phase behavior — meridional dipole-like pattern

**SST:** Traveling wave from region south of Cape Hatteras northward till Iceland — reaches after full cycle Great Britain

► High anomalies in TAUX correspond to low anomalies in SST — cooling effect in particular visible in northern part
Interannual variability in SODA

Covarying phase propagation patterns in SST + SSH

(a) TAUX
(b) SSH
(c) SST

(d) TAUX
(e) SSH
(f) SST

(g) TAUX
(h) SSH
(i) SST
Interannual variability in SODA

Oscillatory patterns at different depth

1. Separate M-SSA analysis of temperature at various depths
2. Rotation onto $\text{TAUX} + \text{SSH} + \text{SST}$ solution
3. Calculation of phase coherence index

- **7.7-yr**: stable ST-EOF pattern over depth
  - Strong equivalent-barotropic component
- **12.7-yr & 2.7-yr**: barotropic component much weaker
- **All three modes**: Cape Hatteras and Grand Banks region show strong equivalent-barotropic component
Intrinsic vs. external variation

- Observed variability in the basin-scale circulation due to
  - changes in the external atmospheric forcing or
  - system’s intrinsic instability and nonlinearity?

Rotation of SST+SSH solution onto TAUX solution --- 7.7-yr mode

North Stronger link between TAUX oscillatory mode and SST+SSH oscillatory mode; here TAUX is strong

GSR SST+SSH oscillatory mode less correlated with TAUX oscillatory mode
Intrinsic vs. external variation

Rotation of SST+SSH solution onto TAUX solution --- 7.7-yr mode

SSH rotated – SSH

Var SST

Var SSH

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Conclusions

- Interannual variability in both; wind forcing and ocean dynamics
- Meridional dipole-like structure in atmosphere
- Traveling wave in 7.7-yr mode with strong equivalent-barotropic component
  - High anomalies in wind forcing correspond to low anomalies in SST
  - Results in northern part of North Atlantic indicate stronger link to wind forcing
  - Results in Gulf Stream region indicate more intrinsic variability
- Covarying pattern in SSH and SST in 7.7-yr mode
- In all three modes (7.7-yr, 12.7-yr and 2.7-yr) covarying pattern in SSH and SST in Gulf Stream region
M-SSA


Ocean and atmosphere