Dynamics of Eddies Generated by Sea Ice Leads

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ABSTRACT

Interaction between the atmosphere and ocean in sea ice-covered regions is largely concentrated in leads, which are long, narrow openings between sea ice floes. Refreezing and brine rejection in these leads injects salt that plays a key role in maintaining the polar halocline. The injected salt forms dense plumes that subsequently become baroclinically unstable, producing submesoscale eddies that facilitate horizontal spreading of the salt anomalies. However, it remains unclear which properties of the stratification and leads most strongly influence the vertical and horizontal spreading of lead-input salt anomalies. In this study, the spread of lead-injected buoyancy anomalies by mixed layer and eddy processes are investigated using a suite of idealized numerical simulations. The simulations are complemented by dynamical theories that predict the plume convection depth, horizontal eddy transfer coefficient and eddy kinetic energy as functions of the ambient stratification and lead properties. It is shown that vertical penetration of buoyancy anomalies is accurately predicted by a mixed layer temperature and salinity budget until the onset of baroclinic instability (~3 days). Subsequently, these buoyancy anomalies are spread horizontally by eddies. The horizontal eddy diffusivity is accurately predicted by a mixing length scaling, with a velocity scale set by the potential energy released by the sinking salt plume and a length scale set by the deformation radius of the ambient stratification. These findings indicate that the intermittent opening of leads can efficiently populate the polar halocline with submesoscale coherent vortices with diameters of around 10 km, and provide a step toward parameterizing their effect on the horizontal redistribution of salinity anomalies.

1. Introduction

In polar regions, the exchange of mass, momentum, and heat in the atmosphere-ocean boundary layer is strongly influenced by the sea-ice cover (Goosse et al. 2018; Timmermans and Marshall 2020). Sea ice cover insulates the ocean mixed layer from the atmosphere, and surface heat and salt fluxes are largely confined to gaps in the sea ice such as polynyas and leads (Smith et al. 1990; Morison et al. 1992; Ohshima et al. 2013). This study focuses on leads, which are long, narrow gaps between ice floes with widths ranging from meters to several kilometers, and which last a few hours to several days from formation to freezing (Smith et al. 1990). These openings expose the relatively warm ocean mixed layer to the polar atmosphere, leading to heat loss and re-freezing at the ocean surface.

In the Arctic, the heat flux through leads is estimated to be approximately equal to the heat flux through the rest of the pack ice cover, despite comprising only 1 to 10% of the ice open water fraction (Smith et al. 1990; Morison et al. 1992; Smith et al. 2002). The heat flux is predominantly due to the latent heat of fusion during ice formation (since the mixed-layer underneath the ice is near freezing year-round). Leads have also been shown to account for a large fraction of the total salt flux injected into the mixed-layer in the Arctic (Morison et al. 1992). The salt flux is produced at the ocean surface due to brine rejection generated within the freezing lead. Parameterizing these fluxes in GCMs has been shown to be important in order to accurately simulate the Arctic stratification (Steiner et al. 2004; Nguyen et al. 2009).

The intense, localized, line-shaped buoyancy anomalies produced by leads have dynamical implications throughout the polar regions. The formation of sea-ice within freezing leads creates dense convective plumes that deepen the mixed layer locally, and which can cumulatively modify the mixed layer of the entire sea ice-covered region (Smith and Morison 1998). In buoyancy-dominated regimes, such as for slow-moving leads, the resulting dense plumes penetrate the pycnocline (Smith and Morison 1998) and undergo baroclinic instability, forming submesoscale coherent vortices (SCVs) that spread lead-injected heat and salt anomalies laterally (Bush and Woods 1999; Marshall and Schott 1999; Smith et al. 2002).

In the Arctic, previous studies have observed anticyclonic eddies with cold/high salinity cores near the pycnocline, and horizontal scales of 10 to 20 km (D’Asaro 1988; Muench et al. 2000; Timmermans et al. 2008; Zhao et al. 2014, 2016). Depending on their origin, it has been suggested that these eddies can be generated by instabilities in the shelf break or coastal currents (D’Asaro 1988; Manley and Hunkins 1985; Pickart et al. 2005). Timmermans et al. (2008) showed that eddies observed in the central Canada Basin can be generated by the instabilities in upper-ocean fronts, and are capable of propagating far from their origin.

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An example of a lead eddy generation event from a high-resolution global ocean/sea ice simulation. (top) Daily-averaged sea ice concentration on Dec 12, 2011 in the Weddell Sea sector of the LLC4320 simulation (see Stewart et al. 2018, 2019). Snapshots of (middle row) relative vorticity normalized by the local Coriolis parameter, and (bottom row) potential temperature. These snapshots span the area indicated by the red box in the top panel, at a depth of 352 m.

Much less is known about the presence of SCVs under Antarctic sea ice, and about their relation to the opening leads. Some insight may be gained from high-resolution models, such as the one illustrated in Fig. 1. This simulation was run globally at 1/48° (~1 km grid spacing in polar regions) using the MIT general circulation model (MITgcm), and is described in more detail by Rocha et al. (2016) and Stewart et al. (2019). Fig. 1 shows a Weddell Sea that is populated by many SCVs with diameters of a few tens of km in this model. This figure highlights a specific lead opening event in the southern Weddell Sea, which produces localized convection and a chain of anticyclonic eddies. This mechanism is consistent with previous laboratory experiments (Bush and Woods 1999, 2000) and idealized modeling studies (Smith et al. 2002; Matsumura and Hasumi 2008). However, these eddies occur much deeper (hundreds of meters below the surface) and have consistently larger diameters (around 10 km) than indicated by previous studies. This suggests that such lead
opening events might play a key role in sustaining the eddy field under sea ice. A caveat to investigating the formation of eddies in such models is that even at this fine grid resolution only the largest leads, with widths of several km or more, are resolved.

Many gaps in understanding currently remain regarding lead-generated eddies. In particular, while previous studies have shown that lead eddies spread lead-injected buoyancy anomalies horizontally (Bush and Woods 1999, 2000; Send and Marshall 1995; Matsumura and Hasumi 2008), the length and time scales over which this spread occurs have yet to be quantitatively linked to the properties of the lead and the ambient ocean stratification. Furthermore, while eddy formation beneath the lead has been shown to hinder deepening of the surface mixed layer due to surface buoyancy loss within the lead (Matsumura and Hasumi 2008), a quantitative understanding of this relationship has not yet been established. Previous studies have also not addressed the role of frictional drag against the overlying sea ice in the formation, spread, and longevity of under-lead eddies. Additionally, it remains unclear whether lead-injected buoyancy anomalies are capable of producing larger (tens of km wide) eddies that are sufficiently long-lived to transport buoyancy anomalies over long distances and populate the Arctic and Antarctic polar pycnoclines.

In this study, we use an idealized numerical model with supporting dynamical theories/scalings to investigate the role of lead eddies in mediating the vertical and horizontal spread of lead-injected buoyancy anomalies and their potential contribution to the polar eddy field. The motivation behind this is to extend our current understanding of lead-generated eddy dynamics and assess its potential impact on the stratification and circulation of ice-covered regions, with a particular emphasis on the Antarctic margins. The structure of this article is as follows: In Sect. 2, we present the idealized MITgcm model configuration and the rationale behind our choices of parameters and the parameter space explored. In Sect. 3, we discuss a reference simulation to illustrate the phenomenology of eddy formation beneath leads. We also introduce key eddy-mediated spread of buoyancy anomalies and of the eddies themselves: the mixed-layer depth (MLD) deepening, the lateral buoyancy diffusivity ($\kappa$), and the eddy size ($D$). In Sect. 4, we investigate the dependencies of the MLD, $\kappa$, and $D$ on key lead and stratification parameters: the initial MLD, the pycnocline thickness, the atmospheric temperature, the lead width, the vertical temperature and salinity gradients, and the sea ice drag coefficient. In Sect. 5, we present theoretical predictions for the MLD, $\kappa$, and $D$, and evaluate these theories against our simulation results. In Sect. 6, we summarize our results and provide concluding remarks.

### Table 1: List of parameters used in the reference model run. Italics indicate parameters that are independently varied between model runs.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
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<tr>
<td>$L_x$</td>
<td>50 km</td>
<td>Zonal domain size</td>
</tr>
<tr>
<td>$L_y$</td>
<td>50 km</td>
<td>Meridional domain size</td>
</tr>
<tr>
<td>$H$</td>
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<td>Domain depth</td>
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</tr>
<tr>
<td>$\Delta y$</td>
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<td>Number of simulated days</td>
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<td>3 km</td>
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<tr>
<td>$H_0$</td>
<td>30 m</td>
<td>Initial mixed-layer depth</td>
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<td>Initial pycnocline thickness</td>
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<tr>
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<td>Maximum temperature</td>
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<tr>
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</tr>
<tr>
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<td>Atmospheric Temperature</td>
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<td>$S_i$</td>
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<td>Grid-dependent biharmonic viscosity</td>
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<tr>
<td>$A_r$</td>
<td>1 × 10$^{-3}$ m s$^{-2}$</td>
<td>Vertical Laplacian viscosity</td>
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### 2. Modeling Approach

As lead eddies may play a part in the dynamics of any ice-covered region, we seek to construct a model that represents a range of regional and seasonal profiles characteristic of ice-covered oceans that can be applied both to the Arctic and Antarctic. To allow for such flexibility, we construct a model to permit variations in the ambient ocean stratification, the surface buoyancy forcing, the sea-ice drag, and the width of the lead itself. In Sect. 2a, we introduce the model configuration used and explain the rationale behind the chosen parameters. In Sect. 2b, we introduce a reference simulation that will be explored in Section 3.

#### a. Model Configuration

We implement a model configuration representative of sea-ice covered regions with an idealized lead. Lead dimensions can vary greatly, spanning hundreds of meters to several kilometers wide and tens to hundreds of kilometers long (Smith et al. 1990). Furthermore, sonar data from the Davis Strait indicates that the spacing between leads of ∼1 km widths varies from ∼5 km near the marginal ice-zone to ∼30 km in interior ice regions. These separation distances increase with greater lead widths (Wadhams et al. 1985). An additional constraint on our model is that
the horizontal resolution should sufficiently resolve convection and the eddy formation process beneath the lead. To balance these requirements against the computational cost, we use a domain size of $L_x \times L_y \times H = 50 \text{ km} \times 50 \text{ km} \times 500 \text{ m}$, a uniform horizontal resolution of $\Delta x = \Delta y = 125 \text{ m}$, and a uniform vertical resolution of $\Delta z = 5 \text{ m}$. Our model therefore spans a larger area than previous comparable modeling studies, within a finer grid spacing (e.g., Smith et al. 2002; Matsumura and Hasumi 2008).

We use the MIT general circulation model (MITgcm, Marshall et al. 1997a,b) to solve the nonhydrostatic Boussinesq equations on an $f$-plane (Vallis 2006). We evolve both potential temperature and salinity, which are related to in situ density using the equation of state of Jackett and Mcdougall (1995). We use doubly-periodic horizontal boundary conditions, with a single lead centered at $y = 0$ (see Figs. 2 and 3). Thus, the lead is effectively separated from identical, parallel leads by 50 km in the $y$ direction. We use a free-slip flat bottom boundary $z = -500 \text{ m}$ and impose a rigid lid at the upper boundary $z = 0$. We impose a grid-dependent horizontal biharmonic viscosity in a Laplacian vertical viscosity to control grid-scale accumulation of energy and enstrophy, and we use a flux-limited third-order discrete-space-time scheme to advect potential temperature and salinity.

We prescribe initial conditions of zero velocity and a horizontally uniform potential temperature and salinity stratification comprised of a surface mixed layer overlying a pycnocline, with weak stratification below the pycnocline (see Fig. 2). Specifically, we set $\theta = \theta_0(z)$ and $S = S_0(z)$.

Table 2: List of model experiments. Bolded values indicate parameters that deviate from their reference values. For parameter definitions, refer to Table 1.

<table>
<thead>
<tr>
<th>Batch</th>
<th>Experiment</th>
<th>$W$ (km)</th>
<th>$H_0$ (m)</th>
<th>$H_{\text{pyc}}$ (m)</th>
<th>$\theta_{\text{max}}$ (°C)</th>
<th>$S_{\text{max}}$ (psu)</th>
<th>$T_{\text{ atm}}$ (°C)</th>
<th>$r_b$ ($\times 10^5$ m/s)</th>
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<td>34.6</td>
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</tr>
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</table>

Here $\xi = (z + (H_0 + H_{\text{pyc}}/2))/H_{\text{pyc}}$ is a normalized vertical coordinate that is equal to zero in the middle of the pycnocline, $z = -H_0 + H_{\text{pyc}}$, with $H_0$ being the initial mixed layer thickness and $H_{\text{pyc}}$ being the pycnocline thickness. The
vertical structure function
\[\Gamma(z) = \frac{\sqrt{(1-\bar{z})^2 + 4\gamma^2} - \sqrt{(1+\bar{z})^2 + 4\gamma^2}}{\sqrt{1+\gamma}}, \quad (2)\]

with steepness parameter \(\gamma = 0.01\), produces an approximately piecewise-linear stratification, and is plotted in Fig. 2a. The parameters \(\Delta S = S_{\text{max}} - S_{\text{min}}\) and \(\Delta \theta = \theta_{\text{max}} - \theta_{\text{min}}\) define the total changes in potential temperature and salinity across the pycnocline, respectively. The surface temperature is set to the surface freezing temperature, calculated using \(\theta_f = 0.0901 - 0.0575 \cdot S_{\text{min}}\) (Millero and Leung 1976). All of our simulations are initialized from this prescribed stratification, with varying \(H_0\), \(H_{\text{pyc}}\), \(\Delta \theta\), and \(\Delta S\). Each experiment is integrated for a total of 50 days, by which time the lead-input buoyancy anomalies typically reach the northern and southern domain boundaries.

The model is forced at the surface by parameterized growth of ice within a lead of width \(W\), and the flow is damped by a parameterized stress between the ocean and the overlying sea ice, which is assumed to be stationary for simplicity. We do not attempt to simulate sea ice growth and dynamics explicitly because this would introduce substantial additional complexity to the model (Semtner 1976; Hibler III 1979), and our focus is on the oceanic response. We therefore prescribe a time-dependent salt flux \(\hat{\Sigma}(t)\) following Anderson (1961) that is uniform across the width of the lead and depends only on a specified atmosphere-ocean temperature difference parameter, \(T_{\text{atm}}\). Fig. 2c shows this salt flux as a function of time in our reference simulation. We additionally assume that any heat anomalies that reach
the surface within the lead are immediately removed by air-ice-ocean interactions, which we represent by restoring the potential temperature to the freezing temperature in the surface grid boxes over a time scale of one day. We represent ocean-sea ice drag mechanical stresses via a linear drag, with drag velocity $r_{bh}$ that is applied across the entire surface of the model domain.

This model setup allows both 2D and 3D configurations, which allows us to compares experiments with eddies (in the 3D cases) and without eddies (in the 2D cases), which we discuss in Sect. 3. The 2D and 3D configurations differ from one another only in the number of gridpoints: a single gridpoint is used in the $x$-direction to run the model in 2D.

**b. Reference Case**

Previous studies regarding lead-generated eddies have primarily focused on the Arctic (Smith et al. 2002; Matsumura and Hasumi 2008). However, conditions surrounding Antarctica may be more or less favorable for the genesis and longevity of lead eddies due to its deeper pycnocline and colder atmospheric forcing, which motivates us to design a model configuration that is representative of the Antarctic. We specifically target the Weddell Sea, because
the global high-resolution model discussed in Sect. 1 sim-
ulates an abundance of under-ice SCVs in this region
(see Fig. 1). The latitude is therefore set at 70° S with a
Coriolis parameter $f_0 = -1.375 \times 10^{-4}$ s$^{-1}$.

We select our reference case based on the analysis of
measurements from 2859 CTD profiles deployed in the
Weddell Sea continental slope (Hatterman 2018). The
analysis provides the temperature and salinity profiles both
on-shore and 100 km off-shore between the years 1977 and
2016. We choose to favor off-shore conditions as it better
reflects the majority of the region, as well as conditions
that are better representative of all months of the year,
and not a specific season.

Based on these profiles, we select reference parameter
values of $S_{\text{max}} = 34.6$ psu, $S_{\text{min}} = 34.2$ psu, $\theta_{\text{max}} = 0.5^\circ$C,
$H_0 = -30$ m, and $H_{\text{pyc}} = 150$ m. The freezing tem-
perature is calculated using $S_{\text{min}}$, such that $\theta_f \approx -1.9^\circ$C. This
value is comparable with the typical temperatures observed
in the Antarctic (Turner et al. 2005). The atmosphere-
sea temperature difference is set to $T_{\text{air}} = -25^\circ$C, as
Antarctic atmospheric temperatures typically range from
$-10^\circ$C to $-40^\circ$C. We use an ocean-sea ice drag velocity
of $r_b = 2.5 \times 10^{-4}$ m/s, which is approximately consistent
with a quadratic ice-ocean drag coefficient of $C_{\text{io}} = 5 \times 10^{-3}$ and
surface ice-ocean shear of 0.05 m/s (Cole et al. 2014).
While previous studies typically focus on lead widths
smaller than 1 km, our focus here is on the wider leads,
which produce greater heat and salt fluxes. These leads are
therefore more likely to influence the ocean mixed-layer as
well as favor the growth of maintenance of long-lived lead-
generated eddies. Thus, we set a reference case lead width
of $W = 3$ km, as typical observed lead width for these scales
range between 2–4 km (Key et al. 1993). The parameters
used in our reference simulation are listed in Table 1.

A snapshot of the 3D reference simulation at $t = 10$ days
is shown in Fig. 3. The background shadings shows the
density field at $x = -25$ km and $y = -25$ km, displaying a
plume generated by buoyancy fluxes at the surface (Fig. 2c)
and sinking to levels of $z \approx -85$ m. The bottom undulating
purple curve corresponds to the 1027.5 kg m$^{-3}$ potential
density surface, and the upper horizontal slice shows the
vorticity field right below the lead surface ($z = -12.5$ m),
both illustrating the relationship between the lead buoyancy
forcing and the resulting generation and horizontal spread
of the eddy field.

3. Phenomenology of Lead Eddy Formation

In this section, we briefly describe the formation and
phenomenology of lead eddies in our reference case, which
is discussed in more detail in previous studies (Bush and
Woods 1999, Smith et al. 2002, Matsumura and Hasumi
2008). We then describe our approach for quantifying
specific properties of the eddies and the spread of lead-
input buoyancy anomalies.

3.1. Lead Eddy Formation and Phenomenology

The formation of lead eddies begins with the buoyancy
loss due to freezing of the ocean surface-layer, resulting in
a convective plume (Fig. 2b and Fig. 3). Mixing within this
plume creates a deeper and denser mixed layer beneath the
lead. The mixed layer deepens until it reaches its level of
neutral buoyancy and then spreads these buoyancy anoma-
lies horizontally. This process is particularly clear in the
2D simulations (right column) of Fig. 4, which compares
the evolution of 2D and 3D along-lead averaged density
fields at $t = [1, 3, 5, 10, 30, 50]$ days. In contrast to the
2D simulations, the convective plume in the 3D (left col-
umn) stops deepening around day 3. This suggests that the
depenetration of the mixed layer is arrested by the genesis of
eddies, which cannot form in the 2D simulations.

As the mixed layer becomes deeper and denser it forms a
filament that geostrophically adjusts, occurring approxi-
mately over an inertial timescale. This leads to the forma-
tion of along-lead jets (see e.g., Matsumura and Hasumi
2008), as illustrated in Fig. 2h. The jets are associated
with vorticity anomalies: at the base of the mixed layer
positive vorticity occurs within the filament and negative
vorticity occurs along the outer edges of the filament (Fig.
5a). This structure is baroclinically unstable, leading to the
formation of eddies (Fig. 5b, c). The eddies then begin to
interact and merge with one another, growing continually
until the end of the simulation (Fig. 5d, f). Throughout the
evolution, anticyclonic vortices dominate, consistent with
Fig. 1 and previous studies (e.g., Bush and Woods 1999).

Fig. 4 also shows the impact of the growth and spread
of eddies via the evolution of the cross-lead density field.
In the 2D simulation, the lead-driven buoyancy anomalies
are confined to within a few km of the lead itself, while in
the 3D simulation, the horizontal spread of the buoyancy
anomalies is substantially enhanced and is visible as early
as day 5. By day 50, the lead-input buoyancy anomalies
have modified the upper-ocean stratification across the en-
tire model domain. Specifically, outside of the below-lead
region, buoyancy anomalies spread as a layer of thickness
50–100 m, centered around a depth of approximately 70
m. The stratification of this layer is stronger than that of
the initial mixed layer, but weaker than that of the initial
pycnocline stratification.

To summarize, buoyancy loss within the lead serves to
depth the mixed layer locally, but this deepening appears
to be arrested after a few days. Thereafter, the baroclinic
growth and spread of eddies serves to spread buoyancy
anomalies primarily in the horizontal, rather than the ver-
tical.

3.2. Lead Eddy Diagnostics and Properties

We now aim to quantify key properties of lead-generated
eddies and their impact on the upper-ocean stratification.
Fig. 4: Evolution of the density field with and without the action of eddies. Panels show snapshots of the along-lead-averaged potential density fields in (left) three-dimensional and (right) two-dimensional reference simulations at days 1, 3, 5, 10, 30, and 50 (top to bottom). Contour intervals are 0.01 kg m$^{-3}$. 
These properties will serve as the basis of our model parameter exploration in Sect. 4 and the development of our scalings in Sect. 5. Specifically, we quantify the vertical and horizontal penetration of lead-input buoyancy anomalies via the mixed layer depth and the horizontal eddy diffusivity, respectively, and we quantify the dominant lengthscale of the eddies. The mixed-layer depth and eddy diffusivity quantify the effect of leads and eddies on the upper-ocean stratification in the polar regions, while the eddy size provides an indication of the potential contribution of lead eddies to the eddy field in sea ice-covered regions. Another motivation for quantifying these properties is they are all amenable to direct observational estimates, e.g., via hydrographic sampling.

1) Mixed-Layer Depth

Various previous algorithms have been proposed to diagnose MLDs from hydrographic measurements (e.g., Holte and Talley 2009). However, the simplified initial $\theta$ and $S$ profiles imposed in our simulations (see Sec. 2) allow us to tailor a more specific algorithm. Specifically, we find the MLD using the first inflection point of the density profile beneath the lead starting from the surface, i.e.,

$$\frac{\partial^2 \bar{\sigma}^{x,W}}{\partial z^2} \bigg|_{z=z_{ML}} = 0 \quad \text{for } z_{ML} < -25 \text{ m}. \quad (3)$$

Here $\bar{\sigma}^{x,W}$ denotes the potential density averaged over the entire lead area. To find the inflection point, we interpolate $\sigma$ vertically to a finer vertical resolution (0.1 m) using cubic splines, and then compute the discrete second derivative.

Fig. 5: Evolution of the eddy field. Panels show snapshots of the relative vorticity field, normalized by the Coriolis parameter, at the mixed-layer depth of $z \approx -85$ m in our reference simulation.
of the potential density in (3). The density profile above \( z = -0.25 \) m is excluded because it typically contains a density inversion associated with convection within the lead.

In Fig. 6a, we plot profiles of \( \bar{\sigma}_v \) from our 2D and 3D reference simulations at \( t = 0 \), \( t = 3 \), and \( t = 50 \) days, and indicate the MLDs diagnosed using Eq. (3). The results demonstrate that the method successfully diagnoses the MLD throughout the evolution of the simulation. We also tested density threshold- and density gradient threshold-based quantifications of the MLD (see Holte and Talley 2009). While these methods produce quantitatively very similar estimates of the MLD in our reference simulation, we found that they did not generalize accurately across our sensitivity experiments (see Sect. 4).

To further examine the 2D and 3D MLD evolution, we plotted the diagnosed MLD within the lead as a function of time in Fig. 6b. Comparing the 2D and 3D simulations allows us to disentangle the effects of the surface buoyancy fluxes from the onset of baroclinic instabilities on MLD. Notice that the 3D case does not exhibit any significant trend after \( t \sim 4 \) days. On the other hand, there is a significant trend in the 2D case; between days 4 to 25, the plume depth approximately doubles. Thus, the 2D and 3D cases are not only qualitatively different, but their quantitative differences are substantial. These results are consistent with our qualitative inferences from Fig. 4 in Sect. 3.1: the deepening of the mixed layer halts after a few days in the 3D simulation, but persists and weakens with time in the 2D simulation, suggesting that ML deepening is arrested via the genesis of eddies.

In Fig. 6c, we plot the evolution of the depth diagnosed via (3) as a function of both cross-lead distance and time for the 3D reference case. Outside the lead we refer to this depth as the pycnocline depth, as the waters above this depth are not generally well-mixed (see Fig. 4); the vertically averaged buoyancy frequencies inside and outside the lead are \( N = 0.0026 \text{s}^{-1} \) and \( N = 0.0032 \text{s}^{-1} \), respectively. The pycnocline depth is determined at each point in \( y \) using the potential density from Eq. (3) but averaged only in the \( x \)-direction rather than over the entire lead. Fig. 6c shows that the horizontal spread of weakly-stratified waters from the lead, visible in Fig. 4, is associated with a substantial deepening of the pycnocline depth across an increasingly wide span of the model domain. By the end of the simulation (\( t = 50 \) days), the shallow well-mixed layer is replaced with a deep weakly-stratified layer, extending 10–20 km in either direction perpendicular to the lead. Thus, via the action of the eddies, the lead-input buoyancy anomalies substantially alter the upper ocean stratification over an area many times the size of the lead itself. Potential implications of this effect for ice-covered regions of the ocean are discussed in Sect. 6.

### 2) Eddy Diffusivity

Fig. 4 illustrates the horizontal spread of lead-input buoyancy anomalies due to lead eddies, while Fig. 6c shows their impact on the pycnocline depth in the vicinity of the lead. We now quantify this horizontal spread using the depth-integrated buoyancy anomaly, \( \langle \bar{b}^v \rangle \), where the angle brackets denote a vertical integral,

\[
\langle \bar{b}^v \rangle = \frac{1}{L_x} \int_0^{L_x} \frac{1}{H} \int_0^H b \, dz \, dx.
\]

Here \( b \) (m s\(^{-2}\)) is the buoyancy anomaly,

\[
b = -g \rho - \rho |_{t=0} - \rho_{\text{ref}},
\]

where \( g \) is gravity, \( \rho \) is density and \( \rho_{\text{ref}} = 1027 \text{ kg m}^{-3} \) is the reference density. Fig. 6b shows the evolution of \( \langle \bar{b}^v \rangle \), averaged over the lead, as a function of time, and shows that buoyancy anomalies stop accumulating in the lead around the same time as the ML stops deepening. We plot the evolution of \( \langle \bar{b}^v \rangle \) as a function of time and cross-lead distance in Fig. 7a, visualizing the spread of buoyancy anomalies away from the lead. This is consistent with our previous inferences that lead eddies spread buoyancy anomalies horizontally, based on Figs. 4 and 5.

To quantify the rate of this horizontal spread, we define an eddy diffusivity. We start by writing an equation for the evolution of \( \langle \bar{b}^v \rangle \); under the assumption that the buoyancy anomaly is materially conserved, it may be shown that

\[
\frac{\partial \langle \bar{b}^v \rangle}{\partial t} = -\frac{\partial}{\partial y} \langle v' \bar{b}^v \rangle + \bar{B}^v.
\]

Here primes denote deviations from the along-lead average and vertical integral, e.g., \( b^v \equiv b - \langle \bar{b}^v \rangle \), and \( B \) is the downward surface buoyancy flux with units of m\(^2\) s\(^{-3}\). To derive Eq. (6) we have used the fact that \( \langle \bar{b}^v \rangle = 0 \), by volume conservation. This equation states that the along-lead averaged and depth-integrated buoyancy anomaly changes in response to buoyancy fluxes across the ocean surface and in response to cross-lead eddy buoyancy fluxes, \( \langle v' \bar{b}^v \rangle \). We can model these eddy buoyancy fluxes based on the along-lead averaged and depth-integrated buoyancy anomalies using a Fickian diffusion

\[
\langle v' \bar{b}^v \rangle = -\kappa \frac{\partial \langle \bar{b}^v \rangle}{\partial y},
\]

where \( \kappa \) is the eddy diffusivity. This formulation is consistent with the spreading of buoyancy anomalies shown in Fig. 7a, and with widely-used parameterizations of mesoscale (Gent and McWilliams 1990) and submesoscale (Fox-Kemper et al. 2008) eddies in ocean general circulation models.
Fig. 6: Diagnostics of the mixed-layer depth (MLD) in our reference simulation (see Table 1). (a) Lead-averaged potential density profiles from 3D (solid line) and 2D (dotted line) simulations at \( t = 3 \) days and \( t = 50 \) days. Asterisks and circles indicate the diagnosed MLDs for the 3D and 2D case, respectively. (b) Evolution of the MLD, computed using the along- and across-lead averaged density field, in our 3D and 2D reference simulations, and as predicted by our theory (see Sect. 5). Also shown is the depth-integrated buoyancy anomaly in the 3D case, \( \langle \tilde{b}^x \rangle \), averaged over the lead. (c) Evolution of the pycnocline depth in the 3D case, computed using the along-lead averaged density field, as a function of cross-lead distance.

In Fig. 7b, we plot \( \kappa \) computed directly from our 3D reference simulation as a function of time and across-lead distance. To avoid very large spikes in the diffusivity for the purposes of this plot, we have performed a running time-average of both \( \tilde{v}^x \tilde{b}^x \) and \( \langle \tilde{b}^x \rangle \) before using these quantities to calculate \( \kappa \) via Eq. (7). We varied the length of the time-averaging window from 1 day at \( t = 0 – 10 \) days to 10 days at \( t = 20 – 50 \) days, in order to ensure that the time-averaging window consistently spanned multiple eddy turnover timescales. Despite this additional smoothing, the diagnosed \( \kappa \) remains noisy, with values ranging an order of magnitude: from 10 m²/s to over 100 m²/s. This pronounced spatio-temporal variability is linked to variability in the locations and circulations of the eddies, as shown in Fig. 7d. Here, we plot the depth-averaged eddy kinetic energy (EKE), which is defined as

\[
\text{EKE} = \frac{1}{2} \left( \tilde{u}^t \tilde{\kappa}^t + \tilde{\kappa}^t \tilde{u}^t \right)
\]

where \( u \) and \( v \) are the horizontal velocity components and daggers denote deviations from the zonal mean, e.g., \( \tilde{u} = u - \bar{u} \). We discuss the relationship of the \( \kappa \) with EKE further in Sect. 5b.

The high spatio-temporal variability of \( \kappa \) diagnosed using Eq. (7) hinders intercomparison between simulations, for example in our parameter sensitivity experiments (see Sect. 4). We therefore instead derive a bulk estimate of the eddy diffusivity, \( \kappa_\ell \), which is spatially and temporally uniform. For a one-dimensional tracer spreading uni-dimensionally from a release point, analogous to the spreading of depth-integrated buoyancy anomalies in Fig. 7, the tracer may be expected to spread following \( y \sim \sqrt{\kappa t} \) (LaCasce 2008). This scaling, which we derive from the analytical solution of the one-dimensional diffusion equation, has previously been applied to quantify effective diffusivities from ocean tracer observations (e.g., LaCasce et al. 2014). However, in our model, buoyancy is continually lost through the surface of the lead, so this simple scaling for \( \kappa \) does not apply. We therefore ex-
explicitly solve a one-dimensional forced diffusion equation, and optimize the solution to determine the value of $\kappa_\ell$ that maximizes agreement with the buoyancy anomalies derived from our 3D simulations.

Our solution procedure is as follows. We solve Eqs. (6) and (7) prognostically for $\langle \tilde{b}^x \rangle(y, t)$, setting $\kappa = \kappa_\ell$ to be a constant. We simplify the surface buoyancy flux by neglecting the surface restoring of potential temperature to the freezing temperature, such that

$$\overline{\frac{\bar{B}}{B}} = \begin{cases} -g\beta \Sigma(t)/\rho_{ref}, & |y| < W/2, \\ 0, & |y| > W/2, \end{cases}$$

where $\beta = 8 \times 10^{-4} (g/k g)^{-1}$ is the haline contraction coefficient and $\Sigma$ is the surface salt flux in the lead (see Fig. 2c). We discretize the resulting equations using centered finite differences in $y$, with a grid spacing that matches our 3D simulations, and forward Euler time stepping, ensuring that the time step satisfies the CFL (Courant-Friedrichs-Lewy) criterion for the Laplacian diffusion equation. We then repeatedly solve this system numerically for $\langle \tilde{b}^x \rangle(y, t)$ with varying $\kappa_\ell$. We define the optimal $\kappa_\ell$ as that which yields the smallest least-squared difference in $\langle \tilde{b}^x \rangle(y, t)$ between the 1D solution and the 3D simulation, over all points in space and time. In Fig. 7c, we plot $\langle \tilde{b}^x \rangle(y, t)$ derived from our 1D model using parameters for our 3D reference simulation. In this case, the optimal eddy diffusivity is $\kappa_\ell = 29.1$ m$^2$ s$^{-1}$, which produces a small relative root mean square error of 0.6% between the 3D and 1D $\langle \tilde{b}^x \rangle(y, t)$ fields. The close resemblance between Figs. 7a and 7c shows that the 1D model produces an accurate estimate of $\kappa_\ell$.

3) EDDY SIZE

We now quantify the dominant size of the eddies formed in our simulations. SCV sizes can be determined by using eddy tracking methods (Tarshish et al. 2018; Wang et al. 2016). However, the geometric simplicity of our model domain allows us to make a more straightforward estimate of
the dominant eddy scale using a Fourier mode decomposition. Specifically, we define the velocity anomalies relative to an along-lead average as \( u^\dagger = u - \bar{u} \) and \( v^\dagger = v - \bar{v} \), and decompose them as

\[
\begin{align*}
    u^\dagger &= \int \hat{u}(k)e^{i k \cdot \ell} \, dk, \\
    v^\dagger &= \int \hat{v}(k)e^{i k \cdot \ell} \, dk.
\end{align*}
\]

Here, \( k \) is the wavenumber, and \( \hat{u}(k) \) and \( \hat{v}(k) \) are the Fourier components of \( u^\dagger \) and \( v^\dagger \), respectively.

We quantify the dominant eddy size by examining the distribution of EKE (per unit wavenumber) in Fourier space, which is expressed as

\[
\hat{E}(k) = \frac{1}{2} |\hat{u}(k)|^2 + |\hat{v}(k)|^2.
\]

In Fig. 8a (left panel), we plot the spectrum \( \hat{E}(k) \) on days 3, 5, 30, and 50 to visualize the evolution of the energy in wavenumber space. For each plotted day we first averaged the Fourier components \( \hat{u}(k) \) and \( \hat{v}(k) \) in the cross-lead (y) direction, and over a 3-day window. This plot shows that over time, EKE shifts from relatively small scales at day 3, with energy concentrated between wavelengths of 2.5–3.5 km, to much larger scales at day 50, with energy concentrated around wavelengths of 15–25 km. This suggests that an inverse cascade of energy is taking place, as expected of quasi-two-dimensional turbulence (e.g., Vallis 2006).

To quantify the evolution of the dominant energy-containing lengthscale more precisely, we define the centroid wavenumber,

\[
k_c = \left( \frac{\int k \hat{E}(k) \, dk}{\int \hat{E}(k) \, dk} \right).
\]

The vertical dotted lines in Fig. 8a (left panel) indicate \( k_c \) at days 3, 5, 30, and 50. We then define the eddy diameter \( D \) as

\[
D = \frac{2\pi}{k_c}.
\]

In Fig. 8b we plot \( D \) as a function of time throughout our reference simulation. Consistent with our qualitative inference from the energy spectra, the eddies grow nearly linearly from approximately 1.5 km in size at the start of the simulation until \( t \approx 15 \) days. Subsequently, the eddies continue to grow, but with increased temporal variability, and ultimately reach a scale of \( \approx 20 \) km by \( t = 50 \) days. This suggests that eddy-eddy interactions facilitate the growth of eddies to a size that compares with SCVs observed in sea ice-covered regions (e.g., Zhao et al. 2014).

4. Parameter Sensitivity

In Sect. 3 we identified three key quantities of interest in our simulations: the MLD, the eddy diffusivity \( \kappa_e \), and the dominant eddy size \( D \). In this section, we quantify the sensitivity of these three key quantities to various model parameters in order to draw further insights into the dynamics of lead eddy formation. Specifically, we vary the lead width \( W \), the initial mixed-layer depth \( H_0 \), the pycnocline thickness \( H_pyc \), the cross-pycnocline temperature difference \( \Delta \theta \), the cross-pycnocline salinity difference \( \Delta S \), the atmospheric temperature \( T_{atm} \), and the drag velocity \( r_p \).

Most of these parameters were chosen based on their role in setting the strength of the buoyancy forcing and stratification, which were shown to be important to the eddy properties in previous modeling studies (Matsumura and Hasumi 2008; Smith et al. 2002).

The sensitivity experiments and their corresponding parameter values are listed in Table 2. The parameter ranges chosen are intended to span a wide range of conditions in the ocean’s sea ice covered regions, but are perturbed about a reference state that approximates conditions in the southern Weddell Sea (see Sect. 2). The lead width is varied between 1 to 10 km. These lead widths fall within the range that is observed in nature (Key et al. 1993; Lindsay and Rothrock 1995; Reiser et al. 2020) and that can be represented in our model configuration; smaller widths would necessitate higher resolutions, while larger widths would require larger domains. We explore seasonal and regional values of \( H_0 \), ranging from depths as shallow as 10 m, as might occur in the summer, to 200 m, as observed in winters or closer to the shore (Pellichero et al. 2017). Similarly, we examine cases for both shallow and deep pycnoclines, ranging from 50 to 400 m. Vertical temperature and salinity gradients are selected based on observations around Kapp Norvegia (Hattermann 2018). The atmospheric temperature \( T_{atm} \) perturbations are selected based on a typical Antarctic atmospheric temperatures, ranging from -10 °C in the summer to -40 °C in the winter (Chapman and Walsh 2007). Lastly, we vary the sea-ice drag velocity by over an order of magnitude, ranging from 5 \( \times 10^{-3} \) m s\(^{-1}\) and 1.25 \( \times 10^{-3} \) m s\(^{-1}\).

To provide a qualitative sense of the sensitivity of the eddy properties to these parameters, in Fig. 9 we illustrate changes in the model simulations for some of our sensitivity experiments. Specifically, we select cases with \( H_pyc = 50 \) m, \( H_0 = -200 \) m, \( T_{atm} = -40 \) °C, and \( W = 10 \) km because these exhibit some of the most extreme differences from our reference simulation. This figure highlights that changing the initial stratification, the rate of sea ice growth in the lead, and the width of the lead can all qualitatively change the vertical penetration and horizontal spread of buoyancy anomalies, and the character of the resulting eddy field. The sensitivities of the MLD, \( \kappa_e \) and \( D \) are
quantified in Figs. 10 and 11, and are discussed in detail below.

a. Mixed-Layer Depth

In Fig. 10, we present the sensitivity of the MLD deepening, \( \Delta \text{MLD} = \text{MLD} - H_0 \), across all of our sensitivity experiments. The figure indicates that \( \Delta \text{MLD} \) is most sensitive to the parameters governing the ambient ocean stratification (\( H_{\text{pyc}}, H_0, \) and \( \Delta S \)), and to the width of the lead (\( W \)). Our \( H_{\text{pyc}} \) sensitivity experiments produce the largest absolute variation in \( \Delta \text{MLD} \), ranging from \( \sim 25 \) m to 100 m. This figure also shows that \( \Delta \text{MLD} \) decreases from 75 m to 30 m as we increase \( \Delta S \). The effects of both \( H_{\text{pyc}} \) and \( \Delta S \) can be explained via their relationship with stratification: a weaker stratification allows the plume to penetrate deeper into the pycnocline and thereby deepen the mixed-layer depth. Note that, as is the case with lead-driven convection, density is primarily influenced by salinity, and therefore, unlike \( \Delta S \), our selected range of \( \Delta \theta \) very weakly influences the stratification and thus has a negligible effect on \( \Delta \text{MLD} \) (Fig. 10d).

As we increase the initial mixed layer thickness \( H_0 \) from 10 m to 200 m, \( \Delta \text{MLD} \) decreases from \( \sim 75 \) m to 15 m. Qualitatively, this occurs because for a fixed input of buoyancy, a deeper mixed layer leads to a weaker buoyancy anomaly, when vertically averaged over the mixed layer. Thus a deeper mixed layer suppresses the penetration of the convection into the pycnocline. As we increase \( W \) from 1 to 10 km, \( \Delta \text{MLD} \) increases from 45 to 85 m. The explanation for this sensitivity must necessarily go beyond 1-dimensional (vertical) consideration of the mixed layer and pycnocline stratification, suggesting that varying \( W \) changes the eddy formation process and the arrest of the mixed layer deepening. This is discussed further in Sect. 5a.

Surprisingly, Fig. 10 shows that varying \( T_{\text{atm}} \), and thus the surface buoyancy loss, only weakly influences \( \Delta \text{MLD} \). This is discussed further in Sect. 5, where we show that this weak sensitivity is consistent with the theoretical prediction of the MLD based on mixed layer buoyancy budget. The influence of sea ice drag is even weaker, and is negligible in comparison to the other parameters, suggesting that the mechanical interaction of lead eddies with the overlying sea ice does not affect their arrest of the mixed layer deepening.

b. Eddy Diffusivity

In Fig. 10, we also plot the variations of the bulk eddy diffusivity \( \kappa_f \), as diagnosed by our 1D model (Sect. 3b), across our sensitivity experiments. These plots indicate that \( \kappa_f \) is most strongly influenced by lead width, and secondarily by \( H_0 \) and \( \Delta S \).
Our lead width sensitivity experiments produce the largest variations found in $\kappa_{f}$, with values ranging from $\kappa_{f} = 10 \text{ m}^2\text{s}^{-1}$ for a small lead width of $W = 1 \text{ km}$, to $\kappa_{f} = 50 \text{ m}^2\text{s}^{-1}$ for a large lead width of $W = 10 \text{ km}$. Fig. 9 shows that for $W = 10 \text{ km}$ the buoyancy anomalies spread much more rapidly than in our reference case and are larger in magnitude, while the eddies are more intense and more numerous. Taken together, these suggest that the larger net buoyancy loss in wide leads produces a more intense eddy field that spreads buoyancy anomalies more efficiently, and suggests that the largest leads should play an outsized role in modifying the upper-ocean stratification under sea ice.

The eddy diffusivity $\kappa_{f}$ exhibits differing responses to the stratification parameters: over the ranges of $H_0$ and $\Delta S$ explored in our sensitivity experiments, $\kappa_{f}$ increases from $\sim 25 \text{ m}^2\text{s}^{-1}$ to $\sim 45 \text{ m}^2\text{s}^{-1}$. However, $\kappa_{f}$ varies by less than
dependence of the mixed layer deepening 
\[ \Delta \text{MLD} = \text{MLD} - H_0 \] 
(left axis), and bulk buoyancy diffusivity, \( \kappa_\ell \) 
(right axis) to various model configuration parameters. 
We plot sensitivities to: (a) pycnocline thickness \( H_{\text{pyc}} \), (b) 
atmospheric temperature \( T_{\text{atm}} \), (c) initial mixed layer depth \( H_0 \), (d) lead width \( W \), (e) pycnocline temperature gradient \( \Delta \theta \), (f) pycnocline salinity gradient \( \Delta S \) and (g) surface drag coefficient \( r_b \).

These responses are less amenable to interpretation than 
the response of the MLD to the stratification parameters, 
but we will provide further insight into these sensitivities 
via our scaling for \( \kappa_\ell \) in Sect. 5b. Once again, as the 
stratification weakly varies with potential temperature, \( \Delta \theta \) 
has little influence on \( \kappa_\ell \) over the range considered here.

Other parameters that are relatively weaker influences 
on \( \kappa_\ell \) are \( T_{\text{atm}} \) and \( r_b \). Here, \( \kappa_\ell \) increases between 20 to 30 
m\(^2\)s\(^{-1}\) for an increasing \( T_{\text{atm}} \), with little sensitivity below 
\( T_{\text{atm}} = -25 \) °C. In Sect. 5b we show that this is because 
\( \kappa_\ell \) is related to the square root of the EKE, which in turn 
is linearly related to the surface buoyancy loss, and thus 
\( T_{\text{atm}} \). Surprisingly, increasing the sea ice drag has very 
little impact on the eddy field and its effect on spreading 
buoyancy anomalies via \( \kappa_\ell \). This suggests that the lead 
eddy SCVs are largely shielded from direct mechanical 
interaction with the mixed layer, despite forming at the top 
of the pycnocline.

c. Eddy Size

Fig. 9 (right column) suggests that although there are 
substantial differences in the cross-lead migration of eddies 
between simulations, the sizes of the eddies are visually 
similar. Consistent with this observation, we find that 
there is relatively little spread in the eddy size \( D \) early in 
the simulations (\( t \leq 10 \) days), as shown in Fig. 11. Later in 
the simulations the eddy size exhibits substantial intra- and 
inter-simulation variability that largely precludes any firm 
relationships being drawn between \( D \) and our sensitivity 
parameters.

Two notable exceptions are the sensitivity experiments 
with extreme values of \( W \) and \( T_{\text{atm}} \), which are labeled in 
Fig. 11. In these experiments \( D \) typically lies around two 
standard deviations from the mean, suggesting that there 
is a distinguishable increase in \( D \) with both \( W \) and \( T_{\text{atm}} \). 
A speculative interpretation is that stronger net buoyancy 
loss within the lead produces a more energetic eddy field, 
which favors more frequent eddy-eddy interactions and 
thus a more rapid ascent in the dominant energy-containing 
lengthscales of the eddies. However Figs. 9 and 11 show 
that even in these extreme sensitivity experiments, \( D \) ex-
5. Scaling for Under-Lead Eddy Dynamics

To provide further insight into the dynamics of lead eddies, we now pose scalings that relate the MLD, $k_f$, and $D$ to the properties of the lead and the ambient ocean stratification. Note that the scalings developed in this section do not comprise parameterizations, but are rather intended to confer fundamental understandings of the processes occurring beneath leads.

### a. Mixed Layer Depth

To predict the MLD, we first pose a simplified model for the evolution of the MLD in the absence of any lead eddy formation and lateral spreading of buoyancy anomalies. We then show that truncating the model after a few days, around the time that lead eddies are generated, yields an accurate prediction of the long-term MLD in our simulations.

We solve for the MLD, which we denote as $H_m$, at each time $t$ in our simulations. To determine $H_m$, we idealize the stratification as a well-mixed layer for $z > -H_m$, which is identical to the model’s initial stratification defined in Eqs. (1a)–(1b) for $z < -H_m$. More precisely, we assume $\theta$ and $S$ profiles of the form:

$$\theta = \begin{cases} \theta_f, & z > -H_m, \\ \theta_0(z), & z < -H_m, \end{cases}$$

and

$$S = \begin{cases} S^*, & z > -H_m, \\ S_0(z), & z < -H_m. \end{cases}$$

Here the potential temperature of the mixed layer is assumed to be fixed at the surface freezing temperature, $\theta_f$, while the salinity of the mixed layer is an unknown $S^*$. We can relate $S^*$ to $H_m$ by assuming that total salinity is conserved above $z = -H_m$,

$$S^* = \frac{1}{H_m} \left[ \int_{-H_m}^{0} S_0(z) \, dz + \frac{\Sigma}{\rho_0} \right],$$

where $\Sigma$ is again the time-integrated downward surface salt flux in $g/m^2$. These profiles are illustrated for the simple case of piecewise-linear initial $S_0(z)$ and $\theta_0(z)$, profiles in Fig. 12. Note that both $\theta(z)$ and $S(z)$ are discontinuous across the base of the mixed layer.

To determine $H_m$, we assume that the surface buoyancy loss deepens the mixed layer until the stratification at the base of the mixed layer becomes statically stable. The threshold for static stability is crossed when the in situ density difference between the base of the mixed layer and the top of the pycnocline is equal to zero,

$$\rho(S^*, \theta_f, -H_m) = \rho(S_0(-H_m), \theta_0(-H_m), -H_m).$$

### Nonlinearities of the equation of state may play an important role in convective at high latitudes (Akitomo 2005). We therefore determine the in situ density using a simplified nonlinear equation of state that permits both thermobaric and cabbeling effects, similar to that given by (Vallis 2006):

$$\rho(S, \theta, z) = \rho_0 \left[ 1 + \beta (S - S_{\text{ref}}) + \gamma (z - z_{\text{ref}}) - (\alpha_0 + \alpha_z (z - z_{\text{ref}})) (\theta - \theta_{\text{ref}}) - \frac{1}{2} \alpha_\theta (\theta - \theta_{\text{ref}})^2 \right].$$

Here, $\alpha$ is the thermal expansion coefficient (units of K$^{-1}$), $\beta$ is the haline contraction coefficient (units of psu$^{-1}$), and $\gamma$ is the Boussinesq compressibility coefficient (units of m$^{-1}$). The thermal expansion coefficient is allowed to depend on $z$ and $\theta$ to capture the leading nonlinearities due to thermobaricity and cabbeling, respectively:

$$\alpha = -\rho_0^{-1} \partial_\theta \rho = \alpha_0 + \alpha_z (z - z_{\text{ref}}) + \alpha_\theta (\theta - \theta_{\text{ref}}).$$

Here $\alpha_0$, $\alpha_\theta$, and $\alpha_z$ are the reference thermal expansion coefficient, the cabbeling parameter and the thermobaricity parameter, with units of K$^{-1}$, K$^{-2}$ and K$^{-1}$ m$^{-1}$, respectively. The “ref” subscripts denote reference values.
about which the full equation of state has been Taylor-expanded to produce Eq. 17. For each experiment we select $S_{\text{ref}} = S_{\text{min}}$, $\theta_{\text{ref}} = \theta_f$, and $Z_{\text{ref}} = -H_0$. We then calculate the coefficients $a_0$, $b$, $a_\rho$, and $a_\zeta$ using the Gibbs Sea Water Oceanographic Toolbox (McDougall and Barker 2011). The values of these coefficients for the reference case are $a_0 = 2.56 \times 10^{-3} \text{K}^{-1}$, $b = 7.87 \times 10^{-4} \text{psu}^{-1}$, $a_\rho = 1.41 \times 10^{-4} \text{K}^{-2}$, and $a_\zeta = 3.20 \times 10^{-8} \text{K}^{-1} \text{m}^{-1}$.

Substituting Eqs. (14a) – (14b), (15), and (17) into Eq. (16) yields a single nonlinear equation that solves for the mixed-layer depth $H_m$. In Appendix A, we discuss the specific case of a piecewise-linear stratification, which is shown in Fig. 12. This results in a cubic equation for $H_m$, which can be solved to obtain $H_m$ as a function of $H_0$, $H_{\text{pyc}}$, $S_{\text{min}}$, $\theta_f$, $S_{\text{max}}$, $\theta_{\text{max}}$, $\Sigma$ and $\theta_{\text{ref}}$. In Fig. 6b, we plot the predicted $H_m$ as a function of time for our reference case. As our simplified model is one-dimensional, it predicts that the MLD deepens continuously within the lead, whereas in 3D simulations, baroclinic instability halts MLD deepening despite sustained surface buoyancy loss. The 1D model compares favorably with our 2D simulations for the first 10 – 15 days, but then overpredicts the MLD at later times. This is consistent with the modest horizontal spreading of buoyancy anomalies in the 2D simulations shown in Fig. 4, which is not accounted for in our simplified model.

In our simulations, MLD deepening ceases around days 2 to 4, around the time that nonlinear eddies begin to emerge (see Fig. 5). This time scale may be compared with the time scale for exponential growth of unstable baroclinic waves (Eady 1949), $\tau_{\text{Eady}} = \sqrt{Ri} / |f|$. Approximating the Richardson number $Ri$ using the baroclinic shear in the mixed layer and the vertical stratification of the pycnocline, we obtain

$$Ri = \frac{N^2}{V^2} \approx \frac{f^2 \bar{b}_y}{\left( \bar{b}_y \right)_{\text{ML}}} \approx \frac{f^2}{2} \frac{g(a \Delta \theta - b \Delta S)}{H_{\text{pyc}}} \approx \frac{f^2}{H_{\text{pyc}}} \left( \frac{g \beta (S^* - S_{\text{min}})}{\frac{1}{2} \Sigma} \right)^2. \quad (19)$$

For typical values of $S^*$ after one day of ML deepening, we obtain $\tau_{\text{Eady}} \approx 0.3 \text{ days}$. Thus the arrest of the ML deepening occurring several Eady growth timescales later is qualitatively consistent with linear baroclinic instability of the lead convection zone.

Attempting to predict the time at which nonlinear eddies emerge from Eq. (19) is challenging because $S^*$ varies sharply over the first few days of the lead evolution. Therefore, we instead simply assume that the ML deepening is consistently arrested around days 2 to 4 as nonlinear eddies start to form and spread buoyancy anomalies laterally. In Fig. 13a, we compare the predicted $H_m$, averaged between days 2 and 4, with the simulated mixed layer depth in the corresponding 3D simulations, averaged over days 10 to 50. The predicted and diagnosed ML depths agree closely, with a correlation of $r \approx 0.95$.

The most significant shortcoming of our simplified model is that it fails to capture the simulated deepening of the ML with increasing lead width, $W$. This shortcoming may be mitigated using Eq. (19), which suggests that $\tau_{\text{Eady}} \approx W^2$, and thus that $\tau_{\text{Eady}} \approx W$, i.e., for wider leads the ML should deepen for longer before it is arrested by eddy formation.

Fig. 12: Illustrations of idealized piece-wise linear stratification for salinity $S$ (left) and potential temperature $\theta$ (right) used in our theory (Sect. 5a). $S_0(z)$ and $\theta_0(z)$ are the stratifications at $t = 0$ (grey line), and $S(z)$ and $\theta(z)$ are the profiles after lead-induced convection has penetrated into the pycnocline (colored lines).
Fig. 13: Evaluations of our scaling-based predictions of key mixed layer and eddy properties against our 3D simulations. (a) Diagnosed mixed layer depth (MLD), averaged over days 10–50, versus the theoretically predicted mixed layer depth, averaged over days 2 to 4 (see Sect. 5b). (b) Diagnosed bulk buoyancy diffusivity, $\kappa_\ell$ (see Sect. 3b), versus the prediction given by eq. (23). (c) Diagnosed eddy kinetic energy, averaged over the entire volume and duration of the simulation, versus the theoretical upper bound for the EKE at the end of the simulation. The latter is calculated as $\text{EKE}_{\text{ML},H_m}/H$ where $\text{EKE}_{\text{ML}}$ is given by Eq. (21). (d) Eddy size $D$ at $t = 3$ days versus the scaling prediction of Matsumura and Hasumi (2008). In each panel different marker shapes correspond to different parameter variation experiments (see legend), with larger shapes indicating larger absolute values of the corresponding parameter.
b. Eddy Diffusivity

We now propose a scaling for $\kappa_\ell$, in order to quantify the horizontal buoyancy spread due to eddies. Our scaling is based on mixing length theory (Prandtl 1925),

$$\kappa_\ell \sim U_e \ell,$$  \hspace{1cm} (20)

where $U_e$ is an eddy velocity scale and $\ell$ is a mixing length. This formulation serves as the basis of various proposed parameterizations of ocean mesoscale eddies (e.g., Visbeck et al. 1996; Jansen et al. 2015; Wang and Stewart 2020). We anticipate $U_e$ being related to EKE via the relationship $U_e \sim \text{EKE}^{1/2}$. The kinetic energy within the system is all ultimately sourced from the buoyancy loss in the lead, which creates available potential energy (APE, e.g., Winters et al. 1995). Baroclinic instability converts this APE into kinetic energy and sustains the eddy field (Smith et al. 2002; Matsumura and Hasumi 2008).

To develop a scaling for $\kappa_\ell$, in Appendix B we use the energy budget to derive an upper bound on the EKE in the mixed layer, $\text{EKE}_{\text{ML}}$. The upper bound is obtained by assuming (i) that non-conservative terms in the energy budget are negligible, (ii) assuming that all buoyancy anomalies injected at the surface are carried to the base of the mixed layer, (iii) that the energy flux downward across the mixed-layer base is zero, and (iv) that the mean kinetic energy and the vertical EKE are negligible compared to the horizontal EKE, i.e., $\text{KE} \approx \text{EKE}$. This yields the following bound for the horizontally and vertically averaged EKE in the ML:

$$\text{EKE}_{\text{ML}} = \frac{1}{H_m L_x L_y} \int_{-H_m}^{0} dz \int_A \text{EKE} dA \lesssim \frac{B W}{L_y}. \quad (21)$$

Here, $H_m$ is the theoretically-predicted mixed layer depth averaged over days 2 to 4 (see Sect. 5a), and $B = \int_0^t \dot{B} dt$ is the time-integrated upward surface buoyancy flux.

Fig. 13b shows that the EKE diagnosed from our simulations compares very well ($r \approx 0.95$) with the EKE predicted by Eq. (21). The coefficient of proportionality is $C_{\text{EKE}} \approx 0.02$, suggesting that the EKE scales closely with Eq. (21), but that in practice the actual EKE is only a few percent of the theoretical upper bound. Given the weak dependence on surface drag (Fig. 10), this is likely due to energy dissipation in the convective zone beneath the lead.

Conventional scalings of baroclinic eddy fluxes expect the mixing-length scale to vary with drag (Thompson and Young 2006; Larichev and Held 1995). However, since lead eddies appear to be independent of drag, we set the mixing length scale to be the first Rossby radius of deformation imposed by the ambient stratification, i.e., $\ell = R_d$, where

$$R_d = \frac{1}{\pi} \int_0^\infty N dz,$$ \hspace{1cm} (22)

and $N$ is the buoyancy frequency. Thus, Eq. (20) suggests that the mixed layer-averaged $\kappa_\ell$ scales as

$$\kappa_\ell \sim \text{EKE}_{\text{ML}}^{1/2} R_d \sim C_k \left(\frac{W}{L_y} \frac{B}{B} \right)^{1/2} R_d \quad (23)$$

where $\text{EKE}_{\text{ML}}$ is given in equation (21), and $C_k = 0.17$ is the eddy diffusivity scaling coefficient. In Fig. 13(c) we compare these results with $\kappa_\ell$ diagnosed from our 3D simulations, using our 1D model (Sect. 3b), finding a Pearson correlation of $r \approx 0.86$. The largest errors in this scaling result from our experiments with varying pycnocline thickness $H_{\text{pyc}}$ and initial mixed layer depth $H_0$.

c. Eddy Size

A scaling for the eddy size has previously been proposed by Matsumura and Hasumi (2008), so here we test their scaling against our simulations. Matsumura and Hasumi (2008) suggest two scaling regimes for the lead eddy deformation radius $R_e$, depending on its size relative to the lead width $W$. In all of our simulations $R_e \leq W$, so we adopt the scaling

$$R_e \sim \sqrt{\frac{B(t)}{f}}, \quad (24)$$

where $f$ is the Coriolis parameter and $B(t)$ is the time-integrated surface buoyancy flux per unit area.

In Fig. 13d, we compare the eddy size predicted by Eq. (24) with our model results at $t = 3$ days. We chose this time because it occurs earlier than the pronounced divergence in the diagnosed eddy sizes in Fig. 11, and yielded the maximum correlation of $r \approx 0.6$, with a coefficient or proportionality of $\sim 1.2$. Notice that $t = 3$ days coincides with the time at which the MLD deepening is arrested by eddy generation, and is consistent with baroclinic growth over several Eady timescales (see Sect. 5a). This suggests that eddies grow predictably up to $t \sim 3$ days, after which the evolution of the eddy scale becomes dominated by merging events and becomes increasingly chaotic.

In comparison with the correlations found for MLD, $\kappa$ and EKE, Fig. 13d yields a relatively low correlation for $D$. This is in part because we diagnosed $D$ typically only vary by around 10% due to changes in the model parameters (Fig. 10). An exception is that $D$ varies by around 20% in response to changes in the atmospheric temperature $T_{\text{atm}}$, which is captured by the dependence of $B$ on $T_{\text{atm}}$ in Eq. (24)). This may be indicative of sensitivities due to our algorithm for diagnosing eddy sizes, of chaotic behavior in the eddy size evolution even during the initial phase of growth, or of additional dependencies on the model parameters not captured by the theory of Matsumura and Hasumi (2008).
6. Discussion and Conclusion

In this study we developed numerical simulations of (Sect. 2-4) and scalings to explain (Sect. 5) the dynamics of eddies generated by brine rejection and convection within freezing leads. Brine rejection in leads contributes substantially to surface buoyancy fluxes in sea ice covered regions, while eddies serve to horizontally redistribute these localized buoyancy anomalies (Smith et al. 1990; Morison et al. 1992). Previous studies have performed similar idealized numerical simulations to isolate the dynamics of lead eddies, primarily motivated by Arctic conditions (Smith et al. 2002; Matsumura and Hasumi 2008). Our study advances our understanding of lead eddies by posing the simulations in a parameter regime primarily motivated by Antarctic conditions, while also including regimes relevant to the Arctic. We focus on the vertical and lateral spread of lead-input buoyancy anomalies, which lack quantitative understanding (Nguyen et al. 2009), and on the potential for lead eddies to contribute to the under-ice eddy field in polar regions (Zhao et al. 2014).

In Sect. 3a, we showed that our model produces a qualitatively similar phenomenology of lead eddy formation as has been described in previous studies (Bush and Woods 1999, 2000; Smith et al. 2002; Matsumura and Hasumi 2008). Briefly, brine rejection in the lead results in localized convection that deepens the surface mixed layer (see Fig. 4). After 2−4 days, the convective filament becomes baroclinically unstable (see Fig. 5), resulting in the formation of eddies and the arrest of the mixed layer deepening. The eddies then merge, grow, and drift in the cross-lead direction, spreading buoyancy anomalies laterally as they do so (see Fig. 7). We identified three key properties that characterize this eddy formation process, and developed methods to quantify them: the mixed-layer depth (MLD), the bulk eddy diffusivity ($k_\ell$), and the eddy size ($D$). The MLD and $k_\ell$ directly quantify the vertical and horizontal spreading of buoyancy anomalies, respectively, while $D$ allows us to assess the potential for lead eddies to contribute to the under-ice eddy field.

In Sect. 4, we performed a series of sensitivity experiments to determine which properties of the ambient ocean stratification and the buoyancy loss through the lead most strongly influence the MLD, $k_\ell$, and $D$. To complement these experiments, in Sect. 5 we developed scalings relating the MLD, $k_\ell$, and $D$ to the parameters of the lead and the ambient ocean stratification. A synthesis of the key insights from the experiments and scalings is as follows:

- We found that the evolution of the MLD is closely predicted by a mixed layer buoyancy budget (Fig. 13), truncated after 2−4 days to account for the arrest of the mixed layer deepening by eddy formation (Sect. 5a). This time scale is qualitatively consistent with baroclinic growth over an exponential Eady (1949) timescale. Consequently, the MLD is most sensitive to parameters controlling the initial stratification (Fig. 10). However, our scaling does not capture the sensitivity of the MLD to the lead width, $W$ (Fig. 13); in Sect. 5a, we speculate wider leads may reduce the horizontal buoyancy gradients, increasing the bulk Richardson number and thus the Eady growth timescale.

- The bulk eddy diffusivity $k_\ell$ is closely predicted by a mixing length theory-based scaling (Prandtl 1925), with the eddy velocity determined by the eddy kinetic energy (EKE) and the length scale determined by the Rossby radius of deformation ($R_d$) imposed by the ambient stratification. The EKE, in turn, scales with the total potential energy input associated with the buoyancy loss within the lead (Fig. 13b,c). As a result, $k_\ell$ is sensitive to parameters controlling the ambient ocean stratification, which set $R_d$, and $k_\ell$ also varies with the total buoyancy loss in the lead, which is determined by the atmospheric temperature and the lead width (Fig. 10).

- In all of our simulations, $D$ grows nearly linearly at a rate of 0.7 km day$^{-1}$ until $t \sim 10$ days, after which the flow becomes chaotic and the eddies grow to a scale of $\sim 15$ km (Fig. 11). Thus the late-time ($t \geq 10$ days) $D$ is only weakly related to the lead buoyancy loss and the ambient stratification, and is most closely predicted by time since the lead opened. Early in the evolution, the sizes of the eddies are more predictable, showing a modest correlation with the eddy size scaling proposed by Matsumura and Hasumi (2008) at $t = t_{Eady}$. However, the eddy sizes at this time vary little, typically ranging from 2.5 to 4 km, and are very weakly related to the evolution of $D$ at later times. The longevity of the lead eddies and their comparable size to observed SCVs under sea ice (Zhao et al. 2014) suggests that they may contribute to the under-ice eddy field in polar regions.

Across all three of the properties described above, we consistently found that the ocean-ice drag velocity ($r_b$) had a very weak influence. One might expect that mechanical stresses between the ocean and sea ice would draw energy from the eddy field, reducing $k_\ell$ and $D$ and potentially increasing the MLD. The insensitivity to $r_b$, reported here suggests that the vertical structure of eddies shields them from significant mechanical interaction with the sea ice. This is consistent with Meneghello et al. (2021), where it was found that subsurface eddies are shielded from friction at the surface in a parameter regime representative of Arctic leads. In addition, it is also conceivable that this result could change with a more comprehensive representation of the ice-ocean boundary layer and ice-ocean stresses (c.f. Cole et al. 2014; McPhee 2012; Park and Stewart 2016).
These results expand our understanding of under-lead eddy features and the role of leads in upper ocean dynamics in ice-covered regions. However, our idealized modeling approach carries a number of caveats that limit the generalizability of our findings. In particular, we exclude thermodynamics and dynamics of the sea ice, and coupling between the atmosphere and ocean. Previous studies (Kantha 1995; Skyllingstad and Denbo 2001; Heorton et al. 2017) have also indicated that introducing sea-ice velocities influence the mixing and circulation beneath leads. Incorporating these influences might yield more accurate predictions of the convective plume depth and the spreading of buoyancy anomalies due to eddies, and would be particularly interesting to examine these influences on the size and longevity of lead eddies. Furthermore, we do not include the effects of wind within the lead. Winds can herd the ice downwind of the lead and allow for a gradual advancement upward. This process affects the sea-ice growth rate which allows waters to remain at their freezing temperatures, thereby increasing the surface heat and salt fluxes (Bauer and Martin 1983). Another mechanism that can potentially increase these fluxes is the formation of frazil and grease ice (Kantha 1995; Skyllingstad and Denbo 2001; Wilchinsky et al. 2015; Heorton et al. 2017). These mechanisms are not accounted for in our model, and may further increase the surface buoyancy fluxes within the lead.

We also note the large-scale advection for both the mean flow and ice drift within the interior of the Ross/Weddell gyres (typically both reaching a few cm/s, e.g. Garabato et al. 2002 and Holland and Kwok (2012)) are relatively small in comparison to the lead dynamics, and their effects are therefore not considered in this idealized study. However, these assumptions may make our findings less applicable to strong current systems such as the Antarctic Circumpolar Current or Antarctic Slope Current. Similarly, as the regime studied here examines buoyancy-dominating flows with lead widths greater than 1 km, the stress curl across the lead is considered small relative to the lead event. However, these processes may be important to examine when studying different flow regimes or smaller leads (e.g. Bourgault et al. (2020)).

In summary, our findings indicate that lead eddies have the potential to contribute to the eddy field observed in polar haloclines and support scaling relationships of the MLD beneath leads and the horizontal eddy diffusion of density anomalies. These findings provide significant progress towards understanding both the eddy field and the role leads play in upper ocean dynamics in ice-covered regions, and may guide improved parameterizations of the effects of unresolved leads in general circulation models. Further work is required to assess how well these findings apply to coupled sea-ice and ocean model experiments, and to cases in which lead geometries and ice drift patterns are more complex than those considered here. Further work is also required to quantitatively estimate the contribution of eddies to the under-ice eddy field; this will require extensive observations of the eddy field in polar regions, which are particularly lacking around Antarctica.

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APPENDIX A

Theoretical Mixed-Layer Depth

In this appendix we derive an explicit analytical prediction for the mixed layer depth, following the theory posed in Section 5. To derive this prediction we approximate the initial salinity and temperature profiles, \( S_0(z) \) and \( \theta_0(z) \), as piecewise-linear functions of depth:

\[
S_0(z) = \begin{cases} 
S_{\text{min}}, & z_0 \leq z \leq 0, \\
S_{\text{min}}(z-z_{\text{pyc}}) - S_{\text{max}}(z-z_0), & z_{\text{pyc}} \leq z \leq z_0, \\
S_{\text{max}}, & z \leq z_{\text{pyc}},
\end{cases}
\]

and

\[
\theta_0(z) = \begin{cases} 
\theta_f, & z_0 \leq z \leq 0, \\
\frac{\theta_f(z-z_{\text{pyc}}) - \theta_{\text{max}}(z-z_0)}{H_{\text{pyc}}}, & z_{\text{pyc}} \leq z \leq z_0, \\
\theta_{\text{max}}, & z \leq z_{\text{pyc}},
\end{cases}
\]

Here \( z_0 = -H_0 \) corresponds to the base of the mixed layer at \( t = 0 \) and \( z_{\text{pyc}} = -(H_{\text{pyc}} + H_0) \) corresponds to the base of the pycnocline. These piece-wise functions are illustrated in Fig. 12.

We first combine equations (16) and (17) to obtain

\[
\beta(S^* - S_m) - (\alpha + \alpha_z H_m)(\theta_f - \theta_m) \]

\[ - \frac{\alpha_\theta}{2} \left[ (\theta_f - \theta_m)(\theta_f + \theta_m - 2\theta_{\text{ref}}) \right] = 0. \tag{A3}
\]

Here we define \( S_m = S_0(-H_m) \) and \( \theta_m = \theta_0(-H_m) \), and from (15) we have \( S^* = S^*(H_m) \). Thus (A3) is an equation
with a single unknown, $H_m$. To solve, we first write $S_m$ and $\theta_m$ explicitly in terms of $H_m$ using (A1) and (A2):

$$S_m = \frac{S_{\text{min}}(-H_m + H_0 + H_{\text{pyc}}) - S_{\text{max}}(-H_m + H_0)}{H_{\text{pyc}}}, \quad (A4)$$

and

$$\theta_m = \frac{\theta_f (-H_m + H_0 + H_{\text{pyc}}) - \theta_{\text{max}}(-H_m + H_0)}{H_{\text{pyc}}}. \quad (A5)$$

Next, we evaluate the integral in (15) using the initial salinity profile given by (A1) to obtain

$$S^* = S_{\text{min}} + \sum_{\rho_0} \frac{(H_m - H_0)^2}{2H_m} (S_{\text{max}} - S_{\text{min}}). \quad (A6)$$

Finally, we substitute (A4), (A5), and (A6) into (A3) to obtain a cubic polynomial equation for $H_m$,

$$\left[ \alpha_2 + \frac{\alpha_2}{H_0} H_m^2 + \left[ \alpha_1 - \alpha_2 H_0 - 2\alpha_2 - \alpha_3 \right] H_m^3 \right. \left. + H_0 \left[ \alpha_2 - \alpha_1 \right] H_m + \frac{\beta \Sigma}{\rho_0} \frac{H_{\text{pyc}}}{\theta_{\text{max}} - \theta_f} + \alpha_3 H_m^2 \right] = 0. \quad (A7)$$

Here we define constants $\alpha_1 = \alpha + \alpha_\theta (\theta_f - \theta_{\text{ref}})$, $\alpha_2 = \alpha_\theta H_0 (\theta_{\text{max}} - \theta_f)/(2H_{\text{pyc}})$, and $\alpha_3 = \beta(2(S_{\text{max}} - S_{\text{min}}))/(\theta_{\text{max}} - \theta_f)$. Note that we set $\theta_{\text{ref}} = \theta_f$, as discussed in Sect. 5.a. Although (A7) can in principle be solved analytically for $H_m$, the mathematical form of this solution yields little additional physical insight, so in practice we solve (A7) numerically.

**APPENDIX B**

**Constraints on Domain-Averaged Eddy Kinetic Energy**

In this appendix we use the model energy budget to derive an upper bound on the horizontally-averaged eddy kinetic energy. This derivation supports our scalings for the EKE and bulk buoyancy diffusivity in 5.b. We start from the nonhydrostatic Boussinesq momentum equations solved in our simulations,

$$\frac{\partial \mathbf{u}}{\partial t} + (\nabla \times \mathbf{u} + f \hat{z}) \times \mathbf{u} + \nabla \left( \phi + \frac{1}{2} \mathbf{u}^2 \right) = -g \rho / \rho_0 + \mathbf{N}, \quad (B1)$$

where $\mathbf{u}$ is the three-dimensional velocity vector, $f$ is the Coriolis parameter, $\phi = p / \rho_0$ is the dynamic pressure, $b = -g (\rho - \rho_0) / \rho_0$ is the buoyancy, and $\mathbf{N}$ denotes non-conservative effects due to surface drag and viscosity. To obtain an evolution equation for the kinetic energy, we take the vector product of (B1) with $\mathbf{u}$,

$$\frac{\partial}{\partial t} \mathbf{F}_{\text{KE}} = \nabla \cdot \mathbf{F}_{\text{KE}} = \mathbf{b} + \mathbf{N}. \quad (B2)$$

Here $KE = \frac{1}{2} u^2$ is the kinetic energy and $\mathbf{F}_{\text{KE}} = \frac{1}{2} \mathbf{u}^2 + \phi$ is the KE flux. We now consider the kinetic energy budget above an arbitrary vertical level $z = z_0 < 0$. Integrating over the volume $V(z_0)$ enclosed between $z = z_0$ and $z = 0$, we obtain

$$\frac{\partial}{\partial t} \iint_V KEDV = - \iint_V \nabla \cdot \mathbf{F}_{\text{KE}} dV + \iint_V wb \, dV + N$$

$$= \iint_A F^{(z)}_{\text{KE}} dA + \iint_V w b \, dV + N, \quad (B3)$$

where $A$ denotes the horizontal area of the plane $z = z_0$ and $F^{(z)}_{\text{KE}}$ is the vertical component of the KE flux. Here we have rewritten the first term on the right-hand side using the divergence theorem, making use of our domain’s horizontal periodicity and surface boundary condition of zero surface-normal flow. Finally, to obtain an expression for the kinetic energy we integrate (B3) from $t = 0$ to an arbitrary time $t = t_0$,

$$\iint_V KEDV = \int_0^{t_0} dt \int_A F^{(z)}_{\text{KE}} dA$$

$$+ \int_0^{t_0} dt \iint_V wb \, dV + N. \quad (B4)$$

Here we have used the initial condition $\mathbf{KE}|_{z = z_0} = 0$.

The primary source of KE in the model domain is the second term on the right-hand side of (B3), which quantifies the production of KE from potential energy. We now constrain this term by considering the buoyancy budget in our model domain. An approximate evolution equation for the buoyancy is

$$\frac{\partial b}{\partial t} + \nabla \cdot (\mathbf{u} b) \approx - \frac{\partial F_b}{\partial z}. \quad (B5)$$

Here we have assumed that the buoyancy is materially conserved, which is accurate for a buoyancy variable defined using potential density, but omits contributions due to thermobaricity and cabling that would arise if the full in situ density were used to define the buoyancy. We define $F_b$ as the upward surface buoyancy flux, i.e.

$$F_b = \begin{cases} \dot{B}, & z = 0, |\gamma| < W/2, \\ 0, & \text{else}. \end{cases} \quad (B6)$$

We again integrate (B5) over the volume $V(z_0)$ above the level $z = z_0$ to obtain a volume-integrated buoyancy budget,

$$\frac{\partial}{\partial t} \iiint_V b \, dt = - \int_A F_b|_{z = 0} \, dA + \int_A wb|_{z = z_0} \, dA. \quad (B7)$$

Finally, we integrate (B7) from $t = 0$ to $t = t_0$, we obtain

$$\iiint_V b|_{t = 0} - b|_{t = t_0} \, dt = - \int dx \int_{-W/2}^{W/2} B dy$$

$$+ \int_0^{t_0} dt \int_A wb|_{z = z_0} \, dA, \quad (B8)$$
where $B = \int_{0}^{\infty} \bar{b} \, dt$ is the time-integrated upward surface buoyancy flux.

We will now combine (B4) and (B8) to constrain the eddy kinetic energy. First, we use (B8) to quantify the maximum kinetic energy production that can occur at a given depth $z = z_0$. This occurs when $\bar{b}_{t=0} = \bar{b}_{t=0}$, i.e., when the buoyancy anomalies injected at the surface of the lead have all been transported to a depth greater than $z_0$. This implies that

$$\int_{0}^{\infty} dt \int_{A} w \, b \big|_{z=z_0} \, dA \leq BW L_x. \quad \text{(B9)}$$

Second, we substitute (B9) into (B4) to obtain an upper bound on the total kinetic energy. To obtain this bound we consider an extreme case in which (B9) holds for all depths above the base of the mixed layer, i.e., all lead-injected buoyancy anomalies lie at the base of the mixed layer, $z = -H_m$. We further assume that the KE flux downward across the mixed layer base is zero, and neglect the non-conservative terms. Under these assumptions, (B4) becomes

$$\int_{V} KE dV \leq BW L_x H_m. \quad \text{(B10)}$$

Finally, we assume that the mean component of the KE, $\frac{1}{2} \overline{u^2}$, and the vertical eddy kinetic energy, $\frac{1}{2} \overline{w'^2}$, are negligible compared to the horizontal component of the eddy kinetic energy, EKE. Dividing (B10) by the mixed layer depth and the domain horizontal area, we then obtain an upper bound on the horizontally-averaged EKE in the mixed layer

$$\text{EKE}_{\text{M}} = \frac{1}{H_m} \int_{-H_m}^{0} dz \frac{1}{L_x L_y} \int_{A} KE dA \leq \frac{BW}{L_y}. \quad \text{(B11)}$$

References


