A Case Study of Tipping Points: The Wind-Driven Double-Gyre Problem

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Please visit these sites for more info.
http://www.atmos.ucla.edu/tcd/
http://www.environnement.ens.fr/
Motivation

• The **North Atlantic Oscillation (NAO)** is a leading mode of variability of the Northern Hemisphere and beyond.

• It affects **the atmosphere and oceans** on several time and space scales.

• Its **predictive understanding** could help interannual and decadal-scale climate prediction over and around the North Atlantic basin.

• The **hierarchical modeling** approach allows one to give proper weight to the understanding provided by the models vs. their realism, respectively.

• Back-and-forth between **“toy”** (conceptual) and **detailed** (“realistic”) models, and between models and **data**.

Outline, Tipping Points II

- The NAO and the oceans’ wind-driven circulation
- The low-frequency variability of the double-gyre circulation
  - bifurcations in a toy model
    - multiple equilibria, periodic and chaotic solutions
  - some intermediate model results
- Atmospheric impacts
  - simple and intermediate models + GCMs
- Some data analysis – atmospheric and oceanic
- Some very promising NAO results
- Conclusions and bibliography
The North Atlantic Oscillation (NAO)

Positive phase

Negative phase

NAO Index

1860 1880 1900 1920 1940 1960 1980 2000
An example of bifurcations and hierarchical modeling: The oceans’ wind-driven circulation

The mean surface currents are (largely) wind-driven

J. Apel (1987), Principles of Ocean Physics
Kuroshio Extension (KE) Path Changes

Monthly paths from altimeter:
Stable vs. unstable periods

Qiu & Chen (Deep-Sea Res., 2009)
“Limited-contour” analysis for atmospheric low-frequency variability

10-day sequences of subtropical jet paths: blocked vs. zonal flow regimes

Kimoto & Ghil, JAS, 1993a
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  - Natural climate variability: a source of decadal predictability?
The double-gyre circulation and its low-frequency variability

An “intermediate” model of the mid-latitude, wind-driven ocean circulation:
20-km resolution, about 15 000 variables

Shallow-water model

\[
\begin{align*}
\frac{\partial U}{\partial t} + \nabla \cdot (uU) &= -g' h \frac{\partial h}{\partial x} + fV + \alpha A \nabla^2 U - RU - \alpha \tau^x \\
\frac{\partial V}{\partial t} + \nabla \cdot (uV) &= -g' h \frac{\partial h}{\partial y} - fU + \alpha A \nabla^2 V - RV \\
\frac{\partial h}{\partial t} &= -(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y})
\end{align*}
\]

where

\[
U \dot{e}_x + V \dot{e}_y = h \mathbf{u} = h(ue_x + ve_y)
\]

\(g'\): reduced gravity \(\equiv g(\rho_2 - \rho)/\rho\)

\(A\): viscosity coefficient \(\equiv 300 \text{ m}^2\text{s}^{-1}\)

\(R\): Rayleigh coefficient \(\equiv 1/200 \text{ day}^{-1}\)

\(\tau^x\): wind stress \(\tau_0 \cos 2\pi / L (\tau_0 = 1 \text{ dyn cm}^{-2} \text{ & } L = 2000 \text{ km})\)

Reduced gravity (1.5-layer) model

The JJG model’s equilibria

Nonlinear (advection) effects break the (near) symmetry: (perturbed) pitchfork bifurcation?

Subpolar gyre dominates

Subtropical gyre dominates

(Exact) Equilibrium state: \((\alpha_A, \alpha_c) = (1.3, 1.2)\)

- linear case -
  
  \(h(x, y)\)

- nonlinear case -
  
  \(h_n(x, y)\)

2000 km = 20 x

\(\text{curl } \tau^x = 0\)

Multiple equilibria (nonlinear case): \((\alpha_A, \alpha_c) = (1.3, 0.9)\)

- \(h(t=0) = 0\)
  
  \(h(x, y)\)

- \(h(t=0) = h_n(x, y)\)
  
  \(h = \text{ULT} = \text{upper-layer thickness}\)
Time-dependent solutions: periodic and chaotic

To capture space-time dependence, meteorologists and oceanographers often use Hovmöller diagrams.

1. Periodic, with interannual period (2.8 years)

2. Aperiodic (weakly chaotic)
Poor man’s continuation method

Bifurcation diagram

Perturbed pitchfork + Hopf + transition to chaos

Position of Merging Point (km)

$\alpha_A = 1.3$
Interannual variability: relaxation oscillation

0 years  0.4 years  0.8 years  1.2 years

1.6 years  2.0 years  2.4 years  2.8 years
Global bifurcations in “intermediate” models

Bifurcation tree in a QG, equivalent-barotropic, high-resolution (10 km) model: pitchfork, mode-merging, Hopf, and homoclinic
Figure 2. Unfolding of the relaxation oscillations induced by the gyre modes, shown in the plane spanned by the total potential energy of the solution $E_p$ and the difference $\Delta E$ between the subpolar potential energy and the subtropical one (see text for details). The orbits of several limit cycles are

Figure 3. Bifurcation diagram of the highly truncated, four-mode model (5), projected onto the $(A_1 + A_2, A_3)$ plane for $\mu = 1$ and $s = 2$. $P$ stands for pitchfork bifurcation at $\sigma = \sigma_P = 7.61$, while $\sigma = \sigma_{HC} = 10.4299$ at the homoclinic bifurcation. The branches of periodic orbits are replaced by several explicitly computed limit cycles.
The double-gyre circulation: A different rung of the hierarchy

Another “intermediate” model of the double-gyre circulation: slightly different physics, higher resolution – down to 10 km in the horizontal and more layers in the vertical, much larger domain, ...

Bo Qiu, U. of Hawaii, pers. commun., 1997
Model-to-model, qualitative comparison

Bo Qui, 2.5 - layer QG model, 1997

modeled transport ($h_1u_1 + h_2u_2$) along 143°E

Westward recirculation
Model-and-observations, quantitative comparison

Spectra of
(a) kinetic energy of 2.5-layer shallow-water model in North-Atlantic–shaped basin; and
(b) Cooperative Ocean-Atmosphere Data Set (COADS) Gulf-Stream axis data

Figure 7. Comparison between low-frequency variability in an idealized double-gyre model and in observations of the Gulf Stream axis. (a) Spectral results for a 2.5-layer SW model for a basin that approximates the North Atlantic in size and shape, using an idealized wind stress. Maximum
Multi-channel SSA analysis of the UK Met Office monthly mean SSTs for the century-long 1895–1994 interval

Marked similarity with the 7–8-year “gyre mode” of a full hierarchy of ocean models, on the one hand, and with the North Atlantic Oscillation (NAO), on the other: explanation?

Figure 8. Phase composites of the reconstructed 7–8-year SST oscillation. The MSSA window length is 40 year and the contour interval is 0.02°C.
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Atmospheric impact of mid-latitude SST anomalies: A highly contentious issue

- A quasi-geostrophic (QG) atmospheric model in a periodic β-channel, first barotropic (Feliks et al., JAS, 2004; FGS’04), then baroclinic (FGS’07).
- Marine atmospheric boundary layer (ABL), analytical solution.
- Forcing by idealized oceanic SST front.
Vertical velocity at the top of the marine ABL

- The nondimensional $w(H_e)$ is given by

$$w(H_e) = \left[ \gamma \zeta_g - \alpha \nabla^2 T \right],$$

with $\gamma = c_1(f_0 L/U)(H_e/H_a)$ and $\alpha = c_2(g/T_0 U^2)(H_e^2/H_a)$, where $H_a$ is the layer depth of the free atmosphere (\sim 10 km), and $\zeta_g$ the atmospheric geostrophic vorticity.

- Two components: one **mechanical**, due to the geostrophic flow $\zeta_g$ above the marine ABL and one **thermal**, induced by the SST front.
Evolutive spectral analysis

30-day oscillation

70-day oscillation
Simulate atmospheric response to SODA data over the Gulf Stream region

- Use SST (–5 m) data from the SODA reanalysis (50 years)
- Use the FGS’07 QG model in periodic β-channel
  - baroclinic + marine ABL
- Figure shows NAO index:
  - simulated (solid)
  - observed (dashed)
Concluding remarks

- Tipping points and bifurcations: do they really help?
  - Yes, if properly understood and carefully applied!
- Can we predict them?
  - Yes, depending on the problem and the data!
Some references


Reserve slides
Climate models (atmospheric & coupled): A classification

- **Temporal**
  - stationary, (quasi-)equilibrium
  - transient, climate variability

- **Space**
  - 0-D (dimension 0)
  - 1-D
    - vertical
    - latitudinal
  - 2-D
    - horizontal
    - meridional plane
  - 3-D, GCMs (General Circulation Model)
  - Simple and intermediate 2-D & 3-D models

- **Coupling**
  - Partial
    - unidirectional
    - asynchronous, hybrid
  - Full

**Hierarchy:** from the simplest to the most elaborate, iterative comparison with the observational data
Modeling Hierarchy for the Oceans

**Ocean models**

- 0-D: box models – chemistry (BGC), paleo
- 1-D: vertical (mixed layer, thermocline)
- 2-D – meridional plane – THC
  → also 1.5-D: a little longitude dependence
  - horizontal – wind-driven
  → also 2.5-D: reduced-gravity models (n.5)
- 3-D: OGCMs - simplified
  - with bells & whistles (“kitchen sink”)

**Coupled 0-A models**

- Idealized (0-D & 1-D): intermediate couple models (ICM)
- Hybrid (HCM) - diagnostic/statistical atmosphere
  - highly resolved ocean
- Coupled GCM (3-D): CGCM
Forced 7-year cycle in the FGS’04 model

Slow amplitude modulation of 1 °C in the SST front

Low-energy phase

High-energy phase
Spin-up of atmospheric jet

**SST front:**

$L_{oc} = 600 \text{ km}$,

$\Delta T = 3.5 \degree \text{C}$,

$d = 50 \text{ km}$

**Atmospheric jet**

spins up from

$L_a = 2000 \text{ km}$ to

$L_a = 4000 \text{ km}$, much greater speed and strong recirculation
Can we, nonlinear people, help?

The uncertainties might be *intrinsic*, rather than mere “tuning problems”

If so, maybe *stochastic structural stability* could help!
Might fit in nicely with recent taste for “stochastic parameterizations”

*The DDS dream of structural stability* (from Abraham & Marsden, 1978)